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Equilibrium State of an Electron-positron Pair

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Abstract

It is shown that a stable electron-positron pair can exist and be in equilibrium, without recombining. The unlike charges in the electron-positron pair are held apart from each other near the equilibrium position where the attractive Coulomb's force between unlike charges and the repelling force between the spin magnetic moments balance out each other. An electron-positron pair can be considered as an oscillator consisting of two masses on a spring. The calculations of the equilibrium distance and the effective "spring constant" of an electron-positron pair are presented. We suggest that free space might be a network of interacting electrons and positrons (the matter and antimatter particles) bounded together but held from recombining by the combination of the attractive Coulomb's forces and the repelling magnetic forces between electrons and positrons.

1. Introduction

Interaction of charges is described by the Coulomb's law. An electron, with the negative charge -e, and a positron, with a charge +e, apply the attractive electrostatic forces to each other. The electrostatic force, if considered as the only force between an electron and a positron, would cause an acceleration of the electron and the positron in the electron-positron pair toward each other, so the static equilibrium would not be possible. However, electrons and positrons are the particles that have not only charge but also the spin magnetic moment. A magnetic field of one dipole acts on the other magnetic dipole, and the magnetic force can be attractive or repulsive, depending on a mutual orientation of the magnetic moments of the interacting particles. The combination of an attractive electrostatic force and the repulsive force between the magnetic moments can lead to existence of a stable static equilibrium between two charges, when the net force on each charge is zero. Considering the net force between an electron and a positron as the sum of two forces – the electrostatic force between the charges and the magnetic force between the magnetic moments, we can see that these two forces can balance out so the net force on each of two particles will be zero. As a result, two particles with charges and magnetic moments can hold each other from coming closer or moving away from the equilibrium position by the restoring force. Because the properties of electrons and positrons such as their charges, masses, magnetic spin moments, and the binding energy of the electron-positron pair are well known from multiple experimental studies, we can calculate the equilibrium distance between the electron and the positron in the pair. Near the equilibrium, the pair behaves as two masses connected by a spring. Any external force that can cause a change in the distance between the particles in the pair will cause the restoring force toward the equilibrium position. Hence, the pair near the equilibrium can be described using the Hooke's law for a spring where the restoring force near the equilibrium position changes proportionally to the change of the distance from the equilibrium position, with the coefficient of proportionality called the spring constant. The effective "spring constant" for the electron-positron pair near the position of equilibrium can be calculated. In absence of any external forces, the net force between charged particles with the magnetic moments is just the sum of the electrostatic and the magnetic forces between the particles. The gravitational force between the electron and the positron could also be considered but it is by many orders smaller than the electrostatic force so it can be ignored.

While the electron and the positron in an electron-positron pair are at the equilibrium distance from each other, the net force on each particle is zero, the potential energy of the system is negative and at minimum so the equilibrium is stable, and the electron and the positron in the pair cannot come closer to each other and recombine unless enough energy is supplied exceeding the binding energy of the pair. The calculations of the net force, the potential energy, the equilibrium position, and the effective "spring constant" for the electron-positron pair do not require any advanced physics or math beyond the ones considered in the introductory physics courses. The calculations might be very useful for the students in their studies of electricity and magnetism. The conclusions from these calculations might be very far reaching, including the nature of forces and matter in the Universe. This could help students in developing their scientific curiosity and interest in physics.

2. The electron − positron pair in equilibrium.

A positron and electron charges are equal but of opposite sign. According to the Coulomb's law, unlike charges are attracted to each other with the electrostatic force, and an electron and positron, if being initially at rest, would move toward each other and recombine unless some other force, repelling, would prevent them to come into a contact. Such repelling force can be the force between the magnetic moments of the particles. Electrons and positrons do have the spin magnetic moments.

The force between two magnetic moments of infinitely small size (the point magnetic moments) is given by the following equation [1]

$$\vec{F} = \frac{3\mu_0}{4\pi r^4}((\vec{r} \times \vec{M}_1) \times \vec{M}_2 + ((\vec{r} \times \vec{M}_2) \times \vec{M}_1 - 2\vec{r}(\vec{M}_1 \cdot \vec{M}_2) + 5\hat{r}(\vec{r} \times \vec{M}_1) \cdot (\vec{r} \times \vec{M}_2) \quad (1)$$

The potential energy of two point magnetic moments is given in [2] as

$$U = -\frac{\mu_0}{4\pi|r|^3}\left[3(\vec{M}_1 \cdot \hat{r})(\vec{M}_2 \cdot \hat{r}) - \vec{M}_1 \cdot \vec{M}_2\right] - \mu_0 \frac{2}{3}\vec{M}_1 \cdot \vec{M}_2 \delta(\vec{r}) \quad (2)$$

Here $\vec{M}_1$ and $\vec{M}_2$ are the two magnetic moments, $\hat{r}$ is a unit vector in direction from $\vec{M}_1$ to $\vec{M}_2$ and $\delta(\vec{r})$ is the delta-function. The last term with the δ-function vanishes everywhere but the origin and is necessary only to ensure that $\nabla\vec{B}$ vanishes everywhere. If the magnetic moments are parallel to each other and both are either normal or parallel to the vector $\hat{r}$, the force is repulsive [1].

For electrons, the magnitude of the spin magnetic moment is known, $M_{e\,z} = (2.00232)\frac{e}{2m_e}S_z$ where S is the magnitude of the electron spin angular momentum, $S = \sqrt{\frac{3}{4}}\,\hbar$. Positron spin magnetic moment magnitude is the same as for electron, so we can use the equations(1) and (2) for the magnetic force between two equal magnetic spin moments and the Coulomb's Law equation for the electric force (and the corresponding equation for the electrostatic potential energy of two charges) between two equal in magnitude unlike charges. We do not assume though that the permittivity value in the Coulomb's force equation is the permittivity of free space ε_0 because it was determined experimentally by the experiments on "macro" level or calculated from the known values of the permeability and the measured speed of electromagnetic waves in free space, as $\varepsilon_0 = 1/\mu_0 c^2$. As we suggest in this paper, the vacuum might be not a space free of anything, but it might be a network of oscillators (interacting electrons and positrons). That is why we cannot use the macroscopic values of ε_0 and μ_0 in our calculations for the electron-positron pairs . We will evaluate the value of the product $\varepsilon\sqrt{\varepsilon\mu}$ using the known values for the mass, the magnetic spin moment of electron and positron, and the binding energy of an electron-positron pair (e^-e^+ pair).

We can calclathe the equilibrium distance between two charged particles with spin (in this case, in the e^-e^+ pair) by equating the magnitudes of the magnetic and electrostatic forces in the case that they are opposite to each other. For electron and positron, the charges, mass, and the spin magnetic moment magnitude are known from many experiments.

The magnetic force between two magnetic moments is given by equation (1). To simplify the calculations, let us consider the case of $\vec{M}_1$ and $\vec{M}_2$ normal to $\hat{r}$. If $\vec{M}_1$ and $\vec{M}_2$ are parallel or antiparallel, the equation reduces to a simple formula. Direction of the force of $\vec{M}_1$ on $\vec{M}_2$ (repelling or attractive) depends on the magnetic moment vectors being parallel or antiparallel. If the magnetic moments are parallel, the force between them is repulsive, and the magnitude of the force is

$$F_M = \frac{3\mu_0}{4\pi r^4}(|M_1||M_2|) \tag{3}$$

For the case of interacting electron and positron, with $|M_1| = |M_2| = \sqrt{3}\frac{e\hbar}{2m_e}$, the equation (3) becomes

$$F_M = \frac{3\mu}{4\pi r^4}\left(\sqrt{3}\frac{e\hbar}{2m_e}\right)^2 \tag{4}$$

To balance the electrostatic force

$$F_E = \frac{e^2}{4\pi\varepsilon r^2} \tag{5}$$

the two forces must be opposite and of the same magnitude, $F_M = F_E$, so

$$\frac{9\mu}{4\pi r^4}\left(\frac{e\hbar}{2m_e}\right)^2 = \frac{e^2}{4\pi\varepsilon r^2} \tag{6}$$

Solving this equation for *r*, we get the equilibrium distance r_0 between an electron and a positron in a pair as

$$r_0 = 3\sqrt{\varepsilon\mu}\left(\frac{\hbar}{2m_e}\right) \tag{7}$$

In all these equations, we use the permittivity and permeability constants the local values of which can be in general unknown, that is why the subscript 0 is omitted.

The potential energy of the e^-e^+ pair can be calculated as the sum of the electric and magnetic potential energies of the pair,

$$U_{e^-e^+} = -\frac{1}{4\pi\varepsilon}\cdot\frac{e^2}{r} + \frac{3\mu}{4\pi r^3}\left(\sqrt{3}\frac{e\hbar}{2m_e}\right)^2 \tag{8}$$

The binding energy of an e^-e^+ pair is known from the experiments as the energy of a photon that can generate electron-positron pair in free space, $U_{bind\ e^-e^+} = 1.02\ MeV = 1.632\cdot 10^{-13}J$. The binding energy equals the absolute value of the negative potential energy of e^-e^+ pair at the equilibrium distance

$$(U_{e^-e^+})_{r_0} = -\frac{1}{4\pi\varepsilon}\cdot\frac{e^2}{r_0} + \frac{3\mu}{4\pi r_0^3}\left(\sqrt{3}\frac{e\hbar}{2m_e}\right)^2 = -\frac{4}{9}\left(\frac{e^2 m_e}{4\pi\hbar\varepsilon\sqrt{\varepsilon\mu}}\right) \tag{9}$$

From $\left|(U_{e^-e^+})_{r_0}\right| = U_{bind\ e^-e^+}$, we can determine the quantity $\varepsilon\sqrt{\varepsilon\mu}$ as

$$\varepsilon\sqrt{\varepsilon\mu} = \frac{4}{9}\left(\frac{e^2 m_e}{4\pi\hbar U_{bind\ e^-e^+}}\right) = 4.828\cdot 10^{-23}\frac{C^2 s}{Nm^3} \tag{10}$$

and the effective “spring constant” at and near the equilibrium distance as

$$k_{eff} = \left(\frac{d^2U}{dr^2}\right)_{r=r_0} = \frac{2e^2}{4\pi\varepsilon r_0^3} \tag{11}$$

The product $\varepsilon\sqrt{\varepsilon\mu}$ was calculated using only the experimentally measured parameters for electron and positron, - the mass, the charge, the magnetic spin moment, and the binding energy of e^-e^+ pair. The ε and μ values are the local permittivity and permeability (at the distances where the space between particles does not contain anything). If one of these values were measured independently, then the other could be determined using equation (10). For example, if we assume that $\mu = \mu_0 = 4\pi\cdot 10^{-7}\frac{Ns^2}{C^2}$, we get $\varepsilon = 1.227\cdot 10^{-13}\frac{C^2}{Nm^2}$. Then the equilibrium distance between electron and positron In e^-e^+ pair can be evaluated as

$$r_0 = 3\sqrt{\varepsilon\mu}\left(\frac{\hbar}{2m_e}\right) = 6.8\cdot 10^{-14}m \tag{9}$$

Thinking of existence of the stable electron-positron pairs in equilibrium, without recombination, we suggest that vacuum might be in fact an infinite regular network of electrons and positrons interacting via the combination of the electrostatic force between the charges and the force between the magnetic moments of electrons and positrons. Electrons and positrons in free space can be invisible unless some energy (for example a high energy photon) is supplied that breaks the pair apart. The properties of free space as an infinite network of $e^{-}e^{+}$ oscillators can be calculated or evaluated and compared with the known parameters of free space. We hope that such calculations can produce the exact values for both ε and μ . Calculations of normal frequencies of an infinite one-dimensional network of oscillators using the symmetry properties of the network were described by Dr. Yen-Jie Lee in his lecture for the MIT students (available online as a youtube video [3]).

In our parallel paper, we apply the same approach (combination of electrostatic and magnetic forces between particles with charge and magnetic moment) and suggest the models of protons and neutrons.

4. Conclusion

It is shown that combination of the attractive Coulomb's force and the repelling magnetic force between electron and positron leads to possibility for existence of the electron-positron pair in a stable equilibrium state where electron and positron are bounded in a pair. In the pair, an electron and a positron are held at some equilibrium distance from each other, without recombination. The calculations of the equilibrium distance, the potential energy, and the effective "spring constant" of an electron-positron pair are presented. We suggest that free space might be a network of interacting electrons and positrons as particles that have both properties –electric charge and magnetic spin moment.

5. References

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