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Beyond Standard Model: Electrostatic Potential Energy of Quarks, Electron, And Neutrinos As Spinning Composite Structures

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Abstract

The potential energy of any composite structure is related to the binding energy of the structure. The equations for the electrostatic potential energy of quarks, electron-like structures, and neutrinos are presented for our models of elementary particles as spinning composite structures. The structures consist of up to 3 basic elementary charges of magnitude e/3 on the axis of rotation and N charges revolving about the axis. We applied these general equations specifically to the models of different quarks, electron and electron-like particles (muon and tau), and neutral particles (neutrinos). It is shown that the electrostatic potential energies of all considered particles are negative, and the electron’s electrostatic potential energy is the lowest among the considered particles.

1. Introduction: Interaction between elementary particles, basic elementary charges, models of quarks, electron and electron-like particles and neutral particles (neutrinos) as structures composed of basic elementary charges.

In our paper [1], we suggested that the 1st generation elementary particles considered in the Standard Model - an up quark, a down quark, an electron, and a neutral particle - are all spinning composite structures made of just two truly basic elementary particles with electric charges +e/3 and -e/3. In the suggested models, the basic elementary particles have charges of +e/3 and mass m_e/6, but they do not have other intrinsic properties such as spin and magnetic moment. In fact, all suggested structures are composed in a similar way, having some basic elementary charges on the axis of rotation of the structure and the other charges in the structure revolving about that axis. The models can be thought of as the particle structures belonging to the same family (quarks). Each particle modeled as a spinning composite structure has spin due to revolving masses and might have a non-zero magnetic moment due to revolving charges.

In this paper, we present our calculations of electric potential energies of quarks, electron-like structures and neutrinos as spinning composite structures made of several basic elementary charges. The electric potential energy of each rotating structure depends on the distances between the electric charges the composite structure is made of.

In our paper [2], a possible line of rotating structures of “elementary” particles was considered. All our models were composite spinning structures made of just two types of basic elementary particles of charges + e/3 and -e/3. In the structures considered in [2], there were up to three basic charges on the axis of rotation and some number (2 or more) charges revolving about the axis. We considered that all the forces were electromagnetic. The net force on any charge that is on the axis of rotation is the sum of electrostatic forces from all other charges in the structure, and it must be zero. This condition was used in determining the structural relations (form factors) in the rotating geometrical structures. The net force on any revolving charge is the sum of all electromagnetic forces applied to that charge by the other charges in the structure. The net force on the revolving charge must be non-zero and be directed toward the axis of rotation. This condition determines the maximum number of charges revolving about the axis.

The simplest variants of the structures with fractional basic charges are the planar spinning structures considered in [1]:
- a neutral particle (composed of one +e/3 and one -e/3 charges revolving in a circular orbit about the axis normal to the plane of the orbit.
- a planar linear structure with two -e/3 charges revolving about one +e/3 charge (a down quark).
- a planar trigonal structure with three +e/3 charges revolving about one -e/3 charge (an up quark).
- a planar centered square structure with four -e/3 charges revolving about one +e/3 charge (an electron).

In all these structures, with no more than one basic charge on the axis of rotation, the net force on the charge that is on the axis of rotation (if any) is automatically zero due to symmetrical shapes of the structures. The net electromagnetic force on any revolving charge must be non-zero, directed toward the center of the circle.
2. Potential electrostatic energy calculations for planar structures of quarks, an electron, and neutrino.

An electrostatic potential energy of a structure that has one charge \( q_0 \) in the center and \( N \) charges \( q_j \) revolving about the central charge in a circle of radius \( r \) is the sum of the potential energies between each pair of the charges in the structure and can be calculated as

\[
U_N = \frac{1}{4\pi\varepsilon} \sum_{j=1}^{N} q_j \left( q_0 + \sum_{i>j} q_i \frac{q_j}{2\sin \left( \frac{\pi(i-j)}{N} \right)} \right)
\]  

(1)

2.1. Potential energy of the planar neutrino structure.

The planar structure of a neutrino consists of one \(-q = -e/3\) basic charge and one \( q = +e/3 \) basic charge both revolving about the mid-point of the structure in a circle of radius \( r \). The electrostatic potential energy of this structure is

\[
U_d = \frac{q^2}{4\pi\varepsilon} \left( -1 - \frac{1}{2r} \right) = -\frac{1}{2} \frac{q^2}{4\pi\varepsilon} r
\]  

(2)

2.2. Potential energy of the planar d-quark structure.

The planar structure of a d-quark consists of two \(-q = -e/3\) basic charges \( q_1 \) and \( q_2 \) revolving in a circle of radius \( r \) about one \( q = +e/3 \) basic charge \( q_0 \). The electrostatic potential energy of this structure can be calculated from (1) with \( N=2 \) as

\[
U_d = \frac{q^2}{4\pi\varepsilon} \left( -2 + \frac{1}{2\sin \frac{\pi}{2}} \right) = -\frac{3}{2} \frac{q^2}{4\pi\varepsilon} r
\]  

(3)

2.3. Potential energy of the planar u-quark structure.

The planar trigonal structure of a u-quark consists of three \( q = +e/3 \) basic charges \( q_1, q_2, \) and \( q_3 \) revolving in a circle of radius \( r \) about one \(-q = -e/3\) basic charge \( q_0 \). The electrostatic potential energy of this structure is

\[
U_u = \frac{q^2}{4\pi\varepsilon} \left( -3 + \frac{3}{2\sin \frac{\pi}{3}} \right) = -\left( 3 - \sqrt{3} \right) \frac{q^2}{4\pi\varepsilon} r
\]  

(4)

2.4. Potential energy of the planar electron structure.

The planar centered square structure of an electron consists of four \(-q = -e/3\) basic charges \( q_1, q_2, q_3, \) and \( q_4 \) revolving in a circle of radius \( r \) about one \( q = +e/3 \) basic charge \( q_0 \). The electrostatic potential energy of this structure is

\[
U_e = \frac{q^2}{4\pi\varepsilon} \left( -\frac{4}{r} + \frac{4}{r\sqrt{2}} + \frac{2}{2r} \right) = -\left( 3 - 2\sqrt{2} \right) \frac{q^2}{4\pi\varepsilon} r
\]  

(5)

3. Potential electrostatic energy calculations for structures with two charges on the axis of rotation and \( N \) charges revolving about the axis.

The structures with more than one charge on the axis of rotation are 3-D revolving structures of cylindrical symmetry. The two parameters defining the shape of the rotating structure are the orbital radius \( r \) and the distance \( d \) between the charges on the axis. The calculations [2] of the form factor (\( r/d \) ratio) for such structures were done on the condition of zero net force on each charge located on the axis of rotation.

A schematic diagram of such structures is shown in Fig. 1.

Fig. 1. A schematic diagram of a spinning composite structure with two basic elementary charges \( q = +e/3 \) on the axis of rotation and \( N \) basic charges of the opposite sign revolving about the axis.

In the structures, there are two basic charges of the same sign (the charges can be either positive or negative) on the axis of rotation, at the distance 2\( d \) from each other. Several basic charges of the opposite sign revolve about the axis in a circular orbit of radius \( r \). The number \( N \) of revolving charges is specific for the particle. For all the structures to be in a
dynamic equilibrium, the condition of zero net force on each charge located on the axis of rotation must be fulfilled. We assume that the force on each charge at rest is purely electrostatic, and this condition was used in [2] to determine the form factors \((r/d)\) in the structures. The force on each revolving charge is the sum of all the electromagnetic forces acting on that particle. That resulting force must be non-zero and be directed toward the axis of rotation for the particle to move in a circular orbit. That condition determines the maximum number of revolving charges in the structures with two like charges on the axis of rotation. We determined in our previous paper [2] the relation between the geometric parameters (the ratio \(r^2/d^2\) and the form factor \(r/d\)) of a structure with \(N\) revolving charges using the condition of a zero net electrostatic force on any of two like charges on the axis of rotation in a structure with \(N\) revolving charges of opposite sign:

\[
F_{net z} = \frac{q^2}{4\pi\varepsilon(2d)^2} - N \frac{q^2}{4\pi\varepsilon(d^2 + r^2)} \cdot \frac{d}{\sqrt{d^2 + r^2}} = \frac{q^2}{4\pi\varepsilon d^2} \left( \frac{1}{4} - \frac{N}{\left(1 + \frac{r^2}{d^2}\right)^{3/2}} \right) = 0
\]

(6)

The \((r/d)^2\) values of the particles with 2 like charges on the axis and \(N\) revolving charges of the opposite sign were calculated in [2] as

\[
r^2 \frac{d^2} = (4N)^2 - 1
\]

(7)

\[
r \frac{d} = \sqrt{(4N)^2 - 1}
\]

(8)

**Table 1:** \((r/d)^2\) values of the composite particles with 2 like charges on the axis and \(N\) revolving charges of the opposite sign [2]:

<table>
<thead>
<tr>
<th>(N)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>((r/d)^2)</td>
<td>3</td>
<td>4.24</td>
<td>5.35</td>
<td>6.37</td>
</tr>
</tbody>
</table>

Calculated electrostatic potential energy \(U_{2,N}\) of such a structure with 2 charges on the axis of rotation and \(N\) charges revolving about the axis is the sum of the potential energies between each pair of the charges in the structure:

\[
U_{2,N} = \frac{q^2}{4\pi\varepsilon (2d)} + \frac{q^2}{4\pi\varepsilon r} \sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2|\sin\left(\frac{\pi}{N}(i-j)\right)|} - 2N \frac{q^2}{4\pi\varepsilon \sqrt{r^2 + d^2}}
\]

(9)

Using equations (7) and (8) to express \(d\) via \(r\), the equation (9) becomes

\[
U_{2,N} = \frac{1}{4\pi\varepsilon r} \left( \frac{q^2 \sqrt{(4N)^2 - 1}}{2} + \sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2|\sin\left(\frac{\pi}{N}(i-j)\right)|} - 2N \frac{q^2}{\sqrt{1 + \frac{1}{(4N)^2 - 1}}} \right)
\]

(10)

The equation (10) gives the electrostatic potential energy of any structure with two charges on the rotation axis and \(N\) charges of opposite sign revolving about the axis. Let us consider here some structures that can be candidates for the elementary particles listed in the Standard Model.

### 3.1 Neutrino as a spinning structure with two positive basic charges on the axis and two negative basic charges revolving about the axis.

The total charge of this structure is zero, so the structure is neutral - It might be a neutrino. It has 2 charges on the axis of rotation, therefore let us call it \(\nu_2\) and show its composition as the numbers of positive and negative charges of magnitude \(e/3\), as \((\frac{2}{3})\).

As follows from equation (7), the structural parameters \(r\) and \(d\) of such composite particle with \(N = 2\) are related as \((r/d)^2 = 3\) (the form factor of the structure is \(r/d = \sqrt{3}\)). The structure is a rhombus spinning about its short diagonal.

A variant with two negative charges on the axis and two positive charges revolving about the axis has the same arrangement but with the sign of every charge changed to the opposite. This particle can be an antineutrino \(\bar{\nu}_2\), with the
same composition \( \left( \frac{+2}{2} \right) \) but with 2 negative negative charges on the axis and two positive charges revolving about the axis.

The potential energy of \( \tilde{V}_2 \) is the same as the potential energy of \( \nu_2 \). It can be calculated using the equation (10) with \( N = 2 \):

\[
U_{\tilde{V}_2} = U_{2,2} = -\frac{3\sqrt{3}-1}{2} \frac{q^2}{4\pi\varepsilon r} = (-2.10) \frac{q^2}{4\pi\varepsilon r}
\]  

(11)

3.2. A \( d_2 \)-quark as a structure with two positive basic charges on the axis of rotation and three negative basic charges revolving about the axis.

The total charge of this structure is \(-e/3\), and its composition can be shown as \( \left( \frac{+2}{-3} \right) \). It has the same total electric charge as the \( d \)-quark, but unlike the \( d \)-quark model that has one charge on the axis, there are 2 charges on the axis, therefore we call it \( d_2 \)-quark.

The potential energy of \( d_2 \)-quark can be calculated using the equation (10) with \( N = 3 \):

\[
U_{d_2} = U_{2,3} = (-2.635) \frac{q^2}{4\pi\varepsilon r}
\]  

(12)

3.3. A \( u_2 \)-quark as a structure with two negative basic charges on the axis of rotation and four positive basic charges revolving about the axis.

The total charge of this structure is \(+2e/3\), and its composition can be shown as \( \left( \frac{+4}{-2} \right) \). It has the same total electric charge as the \( u \)-quark, but unlike the \( u \)-quark model that has one charge on the axis, there are 2 charges on the axis, therefore we call it a \( u_2 \)-quark.

The potential energy of \( u_2 \)-quark can be calculated using the equation (10) with \( N = 4 \):

\[
U_{u_2} = U_{2,4} = (-2.36) \frac{q^2}{4\pi\varepsilon r}
\]  

(13)

3.4. A muon (\( e_2 \)-particle) as a structure with two positive basic charges on the axis of rotation and five negative basic charges revolving about the axis.

The total charge of this structure is \(-e\), and its composition can be shown as \( \left( \frac{+2}{-5} \right) \). It has the same total electric charge as an electron, but unlike the electron model that has one charge on the axis, there are 2 charges on the axis, therefore we call the muon also a \( e_2 \)-particle.

The potential energy of a muon can be calculated using the equation (10) with \( N = 5 \):

\[
U_{e_2} = U_{2,5} = (-2.05) \frac{q^2}{4\pi\varepsilon r}
\]  

(14)

4. Potential electrostatic energy calculations for structures with three charges on the axis of rotation and \( N \) charges revolving about the axis.

The structures with three charges on the axis of rotation are 3-D revolving structures of cylindrical symmetry. The two parameters defining the shape of the rotating structure are the orbital radius \( r \) and the distance \( d \) between the charges on the axis. The calculations [2] of the form factor (\( r/d \) ratio) for such structures were done on the condition of zero net force on each charge located on the axis of rotation.

A schematic diagram of such structures is shown in Fig. 2.
Fig 2. A structure with 3 like basic charges $q = e/3$ on the axis of rotation and $N$ basic $-q$ charges $q_1, q_2, ..., q_N$ of opposite sign revolving about the axis. In the picture, only 2 revolving charges are shown, but it can be $N$ charges symmetrically distributed around the axis.

The net force on the central charge is zero due to the structure symmetry. The net force on any of two charges (other than the central charge) on the axis of rotation is:

$$|F_{\text{net}}| = \frac{q^2}{4\pi\varepsilon d^2} + \frac{q^2}{4\pi\varepsilon(2d)^2} - N \frac{q^2}{4\pi\varepsilon(d^2+r^2)} \cdot \frac{d}{\sqrt{d^2+r^2}} = \frac{q^2}{4\pi\varepsilon d^2} \left( \frac{5}{4} - \frac{N}{\left( \frac{d^2+r^2}{2} \right)^2} \right) = 0 \quad (15)$$

Solving this equation for structures with $N$ revolving charges, we can determine the form factors of those structures:

$$\frac{r^2}{d^2} = \left( \frac{4N}{5} \right)^2 - 1 \quad (16)$$

$$\frac{r}{d} = \sqrt{\left( \frac{4N}{5} \right)^2 - 1} \quad (17)$$

Calculated electrostatic potential energy $U_{3,N}$ of a structure with 3 like charges on the axis of rotation and $N$ charges of opposite sign revolving about the axis is the sum of the potential energies of each pair of the charges in the structure:

$$U_{3,N} = + \frac{2q^2}{4\pi\varepsilon d} + \frac{q^2}{4\pi\varepsilon(2d)} + \frac{q^2}{4\pi\varepsilon r} \sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2|\sin \left[ \frac{\pi}{N}(i-j) \right]|} - 2N \frac{q^2}{4\pi\varepsilon \sqrt{r^2+d^2}} - N \frac{q^2}{4\pi\varepsilon r} \quad (18)$$

Using equations (16) and (17) to express $d$ via $r$, the equation (18) becomes

$$U_{3,N} = \frac{1}{4\pi\varepsilon r} \left( \frac{5q^2}{\left( \frac{4N}{5} \right)^2 - 1} + \sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2|\sin \left[ \frac{\pi}{N}(i-j) \right]|} - 2N \frac{q^2}{1+ \left( \frac{4N}{5} \right)^2 - 1} - N q^2 \frac{1}{\left( \frac{4N}{5} \right)^2 - 1} \right) \quad (19)$$

We can use equation (19) to calculate the electrostatic potential energy of any structure with three like basic charges on the rotation axis and $N$ basic charges of opposite sign revolving about the axis. Let us consider here the structures with $N = 3$ to 6.

4.1. *Neutrino as a spinning structure with three negative basic charges $-q = -e/3$ on the axis and three positive basic charges $q = +e/3$ revolving about the axis.*

The total charge of this structure is zero, so the structure is neutral - it might be a neutrino. It has 3 charges on the axis of rotation, therefore let us call it $\nu_3$ and show its composition as the numbers of positive and negative charges of magnitude $e/3$, as $\left( +\frac{3}{-3} \right)$. 
As follows from equation (16), the structural parameters r and d of such composite particle with \( N = 3 \) are related as \( \left( \frac{r}{d} \right)^2 = 0.79 \) (the form factor of the structure is \( \frac{r}{d} = 0.89 \)).

A variant with three positive charges on the axis and three negative charges revolving about the axis has the same arrangement but with the sign of every charge changed to the opposite. This particle can be an antineutrino \( \bar{\nu}_3 \), with the same composition \( \left( \frac{+3}{-3} \right) \).

The potential energy of \( \nu_3 \) and \( \bar{\nu}_3 \) is the same and can be calculated using equation (19) with \( N = 3 \):

\[
U_{\nu_3} = U_{\bar{\nu}_3} = (-1.55) \frac{q^2}{4\pi \varepsilon r} \quad (20)
\]

4.2. A \( d_3 \)-quark as a structure with three positive basic charges on the axis of rotation and four negative basic charges revolving about the axis.

The total charge of this structure is \(-e/3\), and its composition can be shown as \( \left( \frac{+3}{-4} \right) \). It has the same total electric charge as the \( d \)-quark, but unlike the \( d \)-quark model that has one charge on the axis, there are 3 charges on the axis, therefore we call it \( d_3 \)-quark.

The potential energy of \( d_3 \)-quark can be calculated using the equation (19) with \( N = 4 \):

\[
U_{d_3} = U_{3,4} = (-3.34) \frac{q^2}{4\pi \varepsilon r} \quad (21)
\]

4.3. A \( u_3 \)-quark as a structure with three negative basic charges on the axis of rotation and five positive basic charges revolving about the axis.

The total charge of this structure is \(+2e/3\), and its composition can be shown as \( \left( \frac{+5}{-3} \right) \). It has the same total electric charge as the \( u \)-quark, but unlike the \( u \)-quark model that has one charge on the axis, there are 3 charges on the axis, therefore we call it \( u_3 \)-quark.

The potential energy of \( u_3 \)-quark can be calculated using the equation (19) with \( N = 5 \):

\[
U_{u_3} = U_{3,5} = (-2.83) \frac{q^2}{4\pi \varepsilon r} \quad (22)
\]

4.4. A \( e_3 \)-particle (tau) as a structure with three positive basic charges on the axis of rotation and six negative basic charges revolving about the axis.

The total charge of this structure is \(-e\), and its composition can be shown as \( \left( \frac{+3}{-6} \right) \). It has the same total electric charge as an electron, but unlike the electron model that has one charge on the axis, there are 3 charges on the axis, therefore we call the tau also a \( e_3 \)-particle.

The potential energy of this particle can be calculated using the equation (19) with \( N = 6 \):

\[
U_{e_3} = U_{3,6} = (-1.30) \frac{q^2}{4\pi \varepsilon r} \quad (23)
\]

| \( \left( \frac{r}{d} \right)^2 \) values of the composite particles with 3 like charges on the axis and \( N \) revolving charges of the opposite sign [4]: |
|---|---|---|---|---|---|
| N  | 2  | 3  | 4  | 5  | 6  |
| \( \left( \frac{r}{d} \right)^2 \) | 0.37 | 0.79 | 1.17 | 1.52 | 1.85 |

5. A symmetrical structure with \( N \) negative charges revolving about the axis and one positive and two negative basic charges on the axis.

The diagram of such structure is shown in Fig 3.
Fig. 3. A diagram of a structure made of one positive basic charge $+q$ in the center of the structure and two negative basic charges $-q$ on the axis of rotation, a distance $d$ from the central charge. The structure is assumed to spin about the axis (a vertical line through the three charges) and the other $N$ charges are revolving about the axis. The figure shows just two revolving charges but, in general, $N$ could be a number $>2$.

The form factors of the structures can be determined from the condition of zero net force on each charge that is on the axis. Due to symmetry of the structure, the net force on the central charge is automatically zero. The condition for a zero net force on each of the other charges on the axis is, for the general case of $N$ revolving negative charges,

$$
|F_{net z}| = +\frac{q^2}{4\pi\varepsilon(2d)^2} - \frac{q^2}{4\pi\varepsilon d^2} + N \frac{q^2}{4\pi\varepsilon(d^2+r^2)} \cdot \frac{d}{\sqrt{d^2+r^2}} = \frac{q^2}{4\pi\varepsilon d^2} \left( -\frac{3}{4} + \frac{N}{\left(1+\frac{r^2}{d^2}\right)^{\frac{3}{2}}} \right) = 0 \quad (24)
$$

From this equation,

$$
\left(\frac{r}{d}\right)^2 = \left(\frac{4}{3}N\right)^{\frac{2}{3}} - 1. \quad (25)
$$

$$
\frac{r}{d} = \sqrt{\left(\frac{4}{3}N\right)^{\frac{2}{3}} - 1} \quad (26)
$$

The results of calculations for different values of $N$ are shown in Table 3.

<table>
<thead>
<tr>
<th>$N$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(r/d)^2$</td>
<td>0.92</td>
<td>1.52</td>
<td>2.05</td>
<td>2.54</td>
<td>3.00</td>
</tr>
</tbody>
</table>

While equation (24) is valid for any $N$, the maximum possible number of revolving charges in the structure must be independently checked on condition that the net force on each revolving charge must be a non-zero central force directed toward the axis of rotation. Evaluated for $N>2$, the electrostatic potential energy is positive, so we can conclude that there are no stable structures with $N>2$ negative charges revolving about the axis where one positive and two negative basic charges reside. The structure with $N = 2$ was considered in [1] as one of possible structure variants of an electron. For this case, $\frac{r}{d} = 0.96$ so the structure is a rhombus spinning about its long diagonal.

The general equation for the electrostatic potential energy $U_{1+2, N}$ of a structure with 1 positive and 2 negative basic charges on the axis of rotation and $N$ negative basic charges revolving about the axis is the sum of the potential energies of
each pair of the charges in the structure:

$$U_{1+2,-N} = -\frac{2q^2}{4\pi\varepsilon d} + \frac{q^2}{4\pi\varepsilon(2d)} + \frac{q^2}{4\pi\varepsilon} \sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2} \left|\sin\frac{\pi N}{2N}(i-j)\right| + 2N \frac{q^2}{4\pi\varepsilon \sqrt{r^2 + d^2}} - N \frac{q^2}{4\pi\varepsilon r}$$  \hspace{1cm} (27)

Using equations (25) and (26) to express d via r, the equation (27) becomes

$$U_{1+2,-N} = \frac{1}{4\pi\varepsilon r} \left( -\frac{3q^2}{\sqrt{\frac{(4N^2)}{3} - 1}} + \frac{\sum_{j=1}^{N} q_j \sum_{i>j}^{N} \frac{q_i}{2} \left|\sin\frac{\pi N}{2N}(i-j)\right|}{\sqrt{\frac{(4N^2)}{3} - 1}} + 2N \frac{q^2}{\sqrt{\frac{1 + \frac{1}{(\frac{4N^2}{3})^{-1}}}} - Nq^2} \right)$$ \hspace{1cm} (28)

where the \( \left( \frac{r^2}{d^2} \right) \) values for the case of N revolving particles are known (see Table 3).

For N = 2, the total charge of this structure is -e, and its composition can be shown as \( \left( \frac{+1}{1} \right) \). It has the same total electric charge as an electron and the same composition of positive and negative basic charges as an electron, but unlike the electron model that has one charge on the axis, there are 3 charges on the axis, and the structure is a rhombus rotating about its long diagonal.

The potential energy of this structure can be calculated using the equation (27) with N = 2:

$$U_{1+2,-2} = (-0.16) \frac{q^2}{4\pi\varepsilon r}$$ \hspace{1cm} (29)

### Conclusion

The general equations for the electrostatic potential energy of quarks, electron-like structures, and neutrinos are presented for our models of elementary particles as spinning composite structures. The structures consist of up to 3 basic elementary charges of magnitude e/3 on the axis of rotation and N charges revolving about the axis. We applied these general equations specifically to the models of different quarks, electron and electron-like particles (muon and tau), and neutral particles (neutrinos). It is shown that the electrostatic potential energies of all considered particles are negative, and the electron’s electrostatic potential energy is the lowest among the considered particles.

### References

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2. Perov, Polievkt, “Beyond Standard Model: Structure Factors of Models of Different Quarks and Neutrinos as Spinning Structures Made of Basic Fractional Charges + e/3” (2024). College of Arts & Sciences Faculty Works. 19. [https://dc.suffolk.edu/cas-faculty/19](https://dc.suffolk.edu/cas-faculty/19)